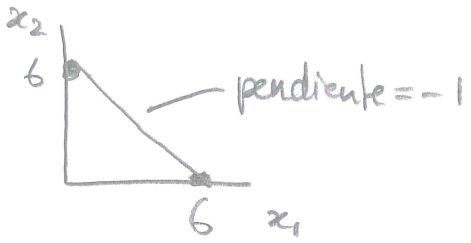
B. 1) a) RP: $p_1x_1 + p_2x_2 = m$

es decir, $2x_1 + 2x_2 = 12$

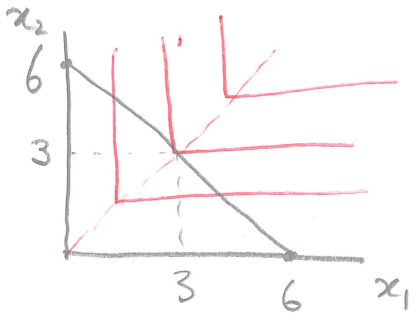


programa

$$\max_{\substack{x_1 \geq 0 \\ x_2 \geq 0}} \min\{x_1, x_2\}$$

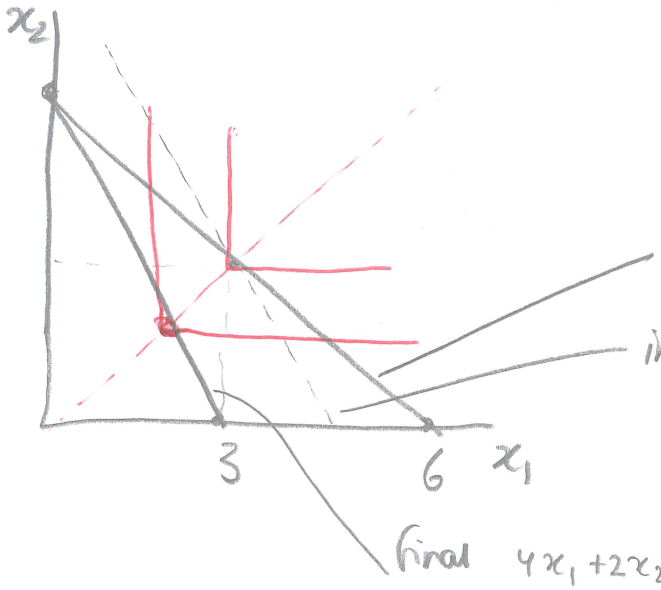
s.a. $2x_1 + 2x_2 = 12$

b)



$\boxed{x_1 = x_2 = 3}$

c)



$\Rightarrow \quad 6x_1 = 12 \Rightarrow \boxed{x_1 = 2 \Rightarrow x_2 = 2}$

Por tanto $ES_1 = ES_2 = 0$ $\qquad ER_1 = ER_2 = 2 - 3 = -1$

B. 2) a) $f(\lambda L, \lambda K) = (\lambda L)^{\frac{a}{3}} (\lambda K)^{\frac{a}{6}} =$

$= \lambda^{\frac{a}{3}} L^{\frac{a}{3}} \lambda^{\frac{a}{6}} K^{\frac{a}{6}} =$

$= \lambda^{\frac{1}{2}a} L^{\frac{a}{3}} K^{\frac{a}{6}} =$

$= \lambda^{\frac{1}{2}a} f(L,K)$

Rendimientos crecientes a escala si y solo si

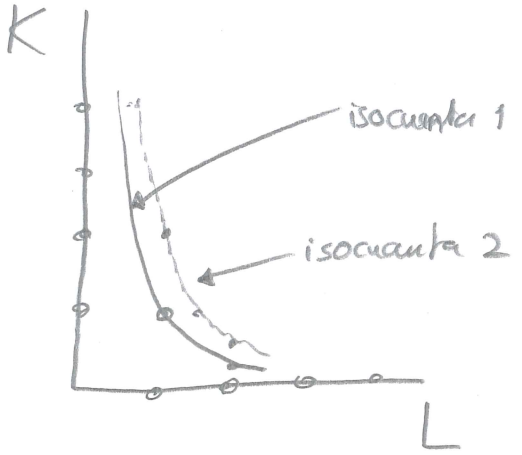
$f(\lambda L, \lambda K) > \lambda f(L,K)$, es decir

$\lambda^{\frac{1}{2}a} f(L,K) > \lambda f(L,K)$ o sea $\frac{1}{2}a > 1$

Por tanto,

f tiene rend. crec. $\Leftrightarrow \frac{1}{2}a > 1 \Leftrightarrow \boxed{a > 2}$

b) $a = 6 \Rightarrow f(L,K) = L^2 K$



isocuanta 1: $L^2K = 1$

$\begin{cases} L=1, K=1 \\ L=2, K=\frac{1}{4} \\ L=\frac{1}{2}, K=4 \end{cases}$

isocuanta 2: $L^2K = 2$

$\begin{cases} L=1, K=2 \\ L=2, K=\frac{1}{2} \\ L=\sqrt{2}, K=1 \end{cases}$

c) pendiente de las isocuantas $= RTS = \dfrac{-\partial f/\partial L}{\partial f/\partial K} = \dfrac{-\frac{a}{3} L^{\frac{a}{3}-1} K^{\frac{a}{6}}}{\frac{a}{6} L^{\frac{a}{3}} K^{\frac{a}{6}-1}} = -2\dfrac{K}{L}$

$\begin{cases} \partial f/\partial L = \frac{a}{3} L^{\frac{a}{3}-1} K^{\frac{a}{6}} \\ \partial f/\partial K = \frac{a}{6} L^{\frac{a}{3}} K^{\frac{a}{6}-1} \end{cases}$

(B.) 2.d) $\alpha = 1 \Rightarrow f(L,K) = L^{\frac{1}{3}} K^{\frac{1}{6}}$

$$\begin{array}{ll} \min & wL + rK \\ L \geq 0 & \\ K \geq 0 & \text{s.a. } L^{\frac{1}{3}} K^{\frac{1}{6}} = y \end{array}$$

Método RTS: $-\frac{w}{r} = RTS \overset{c)}{=} -\frac{2K}{L}$

$$\Rightarrow \frac{w}{r} = 2\frac{K}{L} \Rightarrow wL = 2Kr$$

$$\Rightarrow \begin{cases} L = \frac{2Kr}{w} & (4) \\ K = \frac{wL}{2r} & (5) \end{cases}$$

- $y = L^{\frac{1}{3}} K^{\frac{1}{6}} \overset{(4)}{=} \left(\frac{2Kr}{w}\right)^{\frac{1}{3}} K^{\frac{1}{6}}$

$$\Rightarrow \left(\frac{w}{2r}\right)^{\frac{1}{3}} \cdot y = K^{\frac{1}{2}}$$

$$\rightarrow K = \left[\left(\frac{w}{2r}\right)^{\frac{1}{3}} y\right]^2 = \boxed{y^2 \left(\frac{w}{2r}\right)^{\frac{2}{3}}} \quad (6)$$

- $y = L^{\frac{1}{3}} K^{\frac{1}{6}} \overset{(5)}{=} L^{\frac{1}{3}} \left(\frac{wL}{2r}\right)^{\frac{1}{6}}$

$$\Rightarrow \left(\frac{2r}{w}\right)^{\frac{1}{6}} \cdot y = L^{\frac{1}{2}}$$

$$\Rightarrow L = \left[\left(\frac{2r}{w}\right)^{\frac{1}{6}} y\right]^2 = \boxed{y^2 \left(\frac{2r}{w}\right)^{\frac{1}{3}}} \quad (7)$$

2.e) Costes a largo plazo: $C_p(w,r,y) = wL(w,r,y) + rK(w,r,y) =$

$\overset{(6)+(7)}{=} w y^2 \left(\frac{2r}{w}\right)^{\frac{1}{3}} + r y^2 \left(\frac{w}{2r}\right)^{\frac{2}{3}} =$

$= w^{2/3} 2^{1/3} r^{1/3} y^2 + r^{1/3} w^{2/3} \left(\frac{1}{2}\right)^{\frac{2}{3}} y^2 =$

$= \left(2^{\frac{1}{3}} + \frac{1}{2^{2/3}}\right) y^2 w^{\frac{2}{3}} r^{\frac{1}{3}} =$

$= \left(2^{\frac{1}{3}} \times \left(\frac{2^{2/3}}{2^{2/3}}\right) + \frac{1}{2^{2/3}}\right) y^2 w^{\frac{2}{3}} r^{\frac{1}{3}} =$

$= \boxed{\left(\frac{3}{2^{2/3}}\right) y^2 w^{2/3} r^{1/3}}$

aprox 1.8899

(B.) 2) d) Método Lagrangiano:

$$\mathcal{L}(L,K,\lambda) = wL + rK - \lambda(L^{\frac{1}{3}}K^{\frac{1}{6}} - Y)$$

$$\partial\mathcal{L}/\partial L = 0 \Rightarrow w - \tfrac{1}{3}\lambda L^{-\frac{2}{3}}K^{\frac{1}{6}} = 0 \quad (8)$$

$$\partial\mathcal{L}/\partial K = 0 \Rightarrow r - \tfrac{1}{6}\lambda L^{\frac{1}{3}}K^{-\frac{5}{6}} = 0 \quad (9)$$

$$\partial\mathcal{L}/\partial \lambda = 0 \Rightarrow L^{\frac{1}{3}}K^{\frac{1}{6}} - Y = 0 \quad (10)$$

$$L\times(8) \Rightarrow wL = \tfrac{1}{3}\lambda Y \quad (11)$$

$$K\times(9) \Rightarrow rK = \tfrac{1}{6}\lambda Y \quad (12)$$

$$(11) \Rightarrow L = \frac{\lambda Y}{3w}$$

$$(12) \Rightarrow K = \frac{\lambda Y}{6r} \quad +(10) \Rightarrow \left(\frac{\lambda Y}{3w}\right)^{\frac{1}{3}}\left(\frac{\lambda Y}{6r}\right)^{\frac{1}{6}} = Y$$

$$\Rightarrow \lambda^{\frac{1}{2}} Y^{\frac{1}{2}} (3w)^{-\frac{1}{3}} (6r)^{-\frac{1}{6}} = Y$$

$$\Rightarrow \lambda^{\frac{1}{2}} = \frac{Y}{Y^{\frac{1}{2}}(3w)^{-\frac{1}{3}}(6r)^{-\frac{1}{6}}}$$

$$\Rightarrow \lambda^{\frac{1}{2}} = Y^{\frac{1}{2}}(3w)^{\frac{1}{3}}(6r)^{\frac{1}{6}}$$

$$\Rightarrow \lambda = Y(3w)^{\frac{2}{3}}(6r)^{\frac{1}{3}} \quad (13)$$

Por tanto,

$$L(w,r,Y) = \frac{\lambda Y}{3w} \overset{(13)}{=} \frac{Y(3w)^{\frac{2}{3}}(6r)^{\frac{1}{3}}Y}{3w}$$

$$= Y^2 \frac{r^{\frac{1}{3}}}{w^{\frac{1}{3}}} \times \left(\frac{3^{\frac{2}{3}}6^{\frac{1}{3}}}{3}\right) = \boxed{Y^2\left(\frac{r}{w}\right)^{\frac{1}{3}} \times 2^{\frac{1}{3}}}$$

↑ igual a (7)

y

$$K(w,r,Y) = \frac{\lambda Y}{6r} \overset{(13)}{=} \frac{Y(3w)^{\frac{2}{3}}(6r)^{\frac{1}{3}}Y}{6r}$$

$$= Y^2 \frac{w^{\frac{1}{3}}}{r^{\frac{2}{3}}} \times \left(\frac{3^{\frac{2}{3}}\times 6^{\frac{1}{3}}}{6}\right) = \boxed{Y^2\left(\frac{r}{w}\right)^{\frac{2}{3}} \times 2^{-\frac{2}{3}}}$$

↑ igual a (6)

(B) 2) f) $a=1 \overset{(e)}{\Longrightarrow} C_{1p}(w,r,y)=\left(\frac{3}{2^{2/3}}\right) y^2 w^{2/3} r^{1/3}$

$$\Rightarrow CMe(w,r,y)=\frac{C_{1p}(w,r,y)}{y}=\boxed{\left(\frac{3}{2^{2/3}}\right) y\, w^{2/3} r^{1/3}}$$

$$CMg(w,r,y)=\frac{\partial C_{1p}(w,r,y)}{\partial y}=2\left(\frac{3}{2^{2/3}}\right) y\, w^{2/3} r^{1/3}$$

$$=\boxed{2^{1/3}\times 3\times y w^{2/3} r^{1/3}}$$