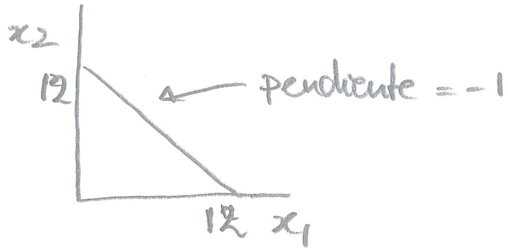
(A.) 1) a) RP: $p_1 x_1 + p_2 x_2 = m$

es decir, $x_1 + x_2 = 12$.

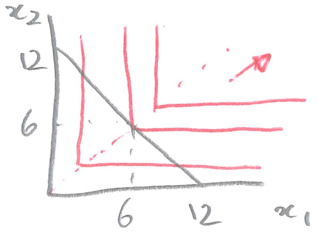


programa:

$$\max_{\substack{x_1 \geq 0 \\ x_2 \geq 0}} \min\{x_1, x_2\}$$

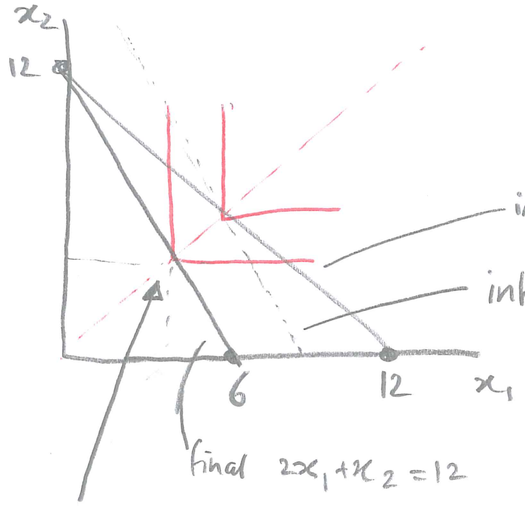
s.a. $x_1 + x_2 = 12$

b)



$\boxed{x_1 = x_2 = 6}$

c)



(x_1, x_2) tal que $2x_1 + x_2 = 12$ y $x_2 = x_1$

$\Rightarrow \quad 3x_1 = 12 \quad \Rightarrow \boxed{x_1 = 4 \Rightarrow x_2 = 4}$

Por tanto $ES_1 = ES_2 = 0$ $\quad ER_1 = ER_2 = 4 - 6 = -2$

ai

(A.) 2) a) $f(\lambda L, \lambda K) = (\lambda L)^{\frac{a}{2}} (\lambda K)^{\frac{a}{4}} =$

$$= \lambda^{\frac{a}{2}} L^{\frac{a}{2}} \lambda^{\frac{a}{4}} K^{\frac{a}{4}} =$$

$$= \lambda^{\frac{3}{4}a} L^{\frac{a}{2}} K^{\frac{a}{4}} =$$

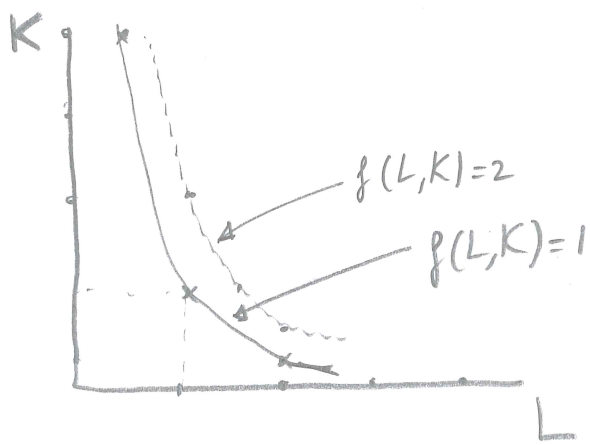
$$= \lambda^{\frac{3}{4}a} f(L,K)$$

Rendimientos crecientes a escala si y sólo si $f(\lambda L, \lambda K) > \lambda f(L,K)$, es decir $\lambda^{\frac{3}{4}a} f(L,K) > \lambda f(L,K)$, o sea $\frac{3}{4}a > 1$.

Por tanto,

f tiene rend. crec. $\iff \frac{3}{4}a > 1 \iff \boxed{a > \frac{4}{3}}$

b) $a = 4 \Rightarrow f(L,K) = L^2 K$



isocuanta 1: $L^2 K = 1$
- $L = 1,\ K = 1$
- $L = 2,\ K = \frac{1}{4}$
- $L = \frac{1}{2},\ K = 4$

isocuanta 2: $L^2 K = 2$
- $L = 1,\ K = 2$
- $L = 2,\ K = \frac{1}{2}$
- $L = \sqrt{2},\ K = 1$

c) pendiente de las isocuantas

$$= RTS = \frac{-\partial f/\partial L}{\partial f/\partial K} = \frac{-\frac{a}{2} L^{\frac{a}{2}-1} K^{\frac{a}{4}}}{\frac{a}{4} L^{\frac{a}{2}} K^{\frac{a}{4}-1}} = -2\frac{K}{L}$$

$$\partial f/\partial L = \frac{a}{2} L^{\frac{a}{2}-1} K^{\frac{a}{4}}$$

$$\partial f/\partial K = \frac{a}{4} L^{\frac{a}{2}} K^{\frac{a}{4}-1}$$

(A.) 2) d) $a=1 \Rightarrow f(L,K) = L^{\frac{1}{2}} K^{\frac{1}{4}}$

$$\min_{\substack{L \geq 0 \\ K \geq 0}} \quad wL + rK$$

$$\text{s.a.} \quad L^{\frac{1}{2}} K^{\frac{1}{4}} = Y$$

Método RTS: $-\frac{w}{r} = RTS \stackrel{c)}{=} -2\frac{K}{L}$

$$\Rightarrow \frac{w}{r} = 2\frac{K}{L} \Rightarrow wL = 2Kr$$

$$\Rightarrow \begin{cases} L = \frac{2Kr}{w} & (4) \\ K = \frac{wL}{2r} & (5) \end{cases}$$

- $Y = L^{\frac{1}{2}} K^{\frac{1}{4}} \stackrel{(4)}{=} \left(\frac{2Kr}{w}\right)^{\frac{1}{2}} K^{\frac{1}{4}}$

$$\Rightarrow \left(\frac{w}{2r}\right)^{\frac{1}{2}} \cdot Y = K^{\frac{3}{4}}$$

$$\Rightarrow K = \left[\left(\frac{w}{2r}\right)^{\frac{1}{2}} Y\right]^{\frac{4}{3}} = \boxed{Y^{\frac{4}{3}} \cdot \left(\frac{w}{2r}\right)^{\frac{2}{3}}} \quad (6)$$

- $Y = L^{\frac{1}{2}} K^{\frac{1}{4}} \stackrel{(5)}{=} L^{\frac{1}{2}} \left(\frac{wL}{2r}\right)^{\frac{1}{4}}$

$$\Rightarrow \left(\frac{2r}{w}\right)^{\frac{1}{4}} \cdot Y = L^{\frac{3}{4}}$$

$$\Rightarrow L = \left[\left(\frac{2r}{w}\right)^{\frac{1}{4}} Y\right]^{\frac{4}{3}} = \boxed{Y^{\frac{4}{3}} \cdot \left(\frac{2r}{w}\right)^{\frac{1}{3}}} \quad (7)$$

2) e) Costes a largo plazo: $C_{lp}(w,r,Y) = wL(w,r,Y) + rK(w,r,Y) =$

$$\stackrel{(6)+(7)}{=} w \cdot Y^{\frac{4}{3}} \cdot \left(\frac{2r}{w}\right)^{\frac{1}{3}} + r\, Y^{\frac{4}{3}} \left(\frac{w}{2r}\right)^{\frac{2}{3}} =$$

$$= Y^{\frac{4}{3}} w^{\frac{2}{3}} r^{\frac{1}{3}} (2)^{\frac{1}{3}} + Y^{\frac{4}{3}} w^{\frac{2}{3}} r^{\frac{1}{3}} \left(\frac{1}{2}\right)^{\frac{2}{3}} =$$

$$= \left(2^{\frac{1}{3}} + \frac{1}{2^{2/3}}\right) \times Y^{\frac{4}{3}} w^{\frac{2}{3}} r^{\frac{1}{3}} =$$

$$= \left(2^{\frac{1}{3}} \times \left(\frac{2^{2/3}}{2^{2/3}}\right) + \frac{1}{2^{2/3}}\right) Y^{\frac{4}{3}} w^{\frac{2}{3}} r^{\frac{1}{3}} = \boxed{\left(\frac{3}{2^{2/3}}\right) Y^{\frac{4}{3}} w^{\frac{2}{3}} r^{\frac{1}{3}}}$$

a3

A. 2) d)

$$\min_{\substack{L\geq 0\\ K\geq 0}} \quad wL + rK \quad \text{s.a. } L^{\frac{1}{2}}K^{\frac{1}{4}} = Y$$

con el Método Langrangiano:

$$\mathcal{L}(L,K,\lambda) = wL + rK - \lambda\left(L^{\frac{1}{2}}K^{\frac{1}{4}} - Y\right)$$

$$\partial\mathcal{L}/\partial L = 0 \Rightarrow w - \tfrac{1}{2}\lambda L^{-\frac{1}{2}}K^{\frac{1}{4}} = 0 \quad (8)$$

$$\partial\mathcal{L}/\partial K = 0 \Rightarrow r - \tfrac{1}{4}\lambda L^{\frac{1}{2}}K^{-\frac{3}{4}} = 0 \quad (9)$$

$$\partial\mathcal{L}/\partial \lambda = 0 \Rightarrow L^{\frac{1}{2}}K^{\frac{1}{4}} - Y = 0 \quad (10)$$

$$L \times (8) \Rightarrow wL = \tfrac{1}{2}\lambda L^{\frac{1}{2}}K^{\frac{1}{4}} = \tfrac{1}{2}\lambda Y \quad (11)$$

$$K \times (9) \Rightarrow rK = \tfrac{1}{4}\lambda L^{\frac{1}{2}}K^{\frac{1}{4}} = \tfrac{1}{4}\lambda Y \quad (12)$$

$$(11) \Rightarrow \boxed{L = \lambda\frac{Y}{2w}}$$

$$(12) \Rightarrow \boxed{K = \lambda\frac{Y}{4r}}$$

$$+ (10) \Rightarrow \left(\lambda\frac{Y}{2w}\right)^{\frac{1}{2}}\left(\lambda\frac{Y}{4r}\right)^{\frac{1}{4}} = Y$$

$$\Rightarrow \lambda^{\left(\frac{1}{4}+\frac{1}{2}\right)} Y^{\frac{1}{2}} (2w)^{-\frac{1}{2}} Y^{\frac{1}{4}} (4r)^{-\frac{1}{4}} = Y$$

$$\Rightarrow \lambda^{\frac{3}{4}} = \frac{Y}{Y^{\frac{1}{2}}(2w)^{-\frac{1}{2}}Y^{\frac{1}{4}}(4r)^{-\frac{1}{4}}}$$

$$= Y^{\frac{1}{4}}(2w)^{\frac{1}{2}}(4r)^{\frac{1}{4}}$$

$$\Rightarrow \lambda = \left(Y^{\frac{1}{4}}(2w)^{\frac{1}{2}}(4r)^{\frac{1}{4}}\right)^{\frac{4}{3}}$$

$$\Rightarrow \lambda = Y^{\frac{1}{3}}(2w)^{\frac{2}{3}}(4r)^{\frac{1}{3}} \quad (13)$$

Por tanto

$$L(w,r,Y) = \lambda\frac{Y}{2w} \overset{(13)}{=} \frac{\left(Y^{\frac{1}{3}}(2w)^{\frac{2}{3}}(4r)^{\frac{1}{3}}\right)Y}{2w}$$

$$= Y^{\frac{4}{3}} \cdot \frac{r^{\frac{1}{3}}}{w^{\frac{1}{3}}} \cdot \frac{2^{\frac{2}{3}}2^{\frac{2}{3}}}{2}$$

$$\Rightarrow \boxed{L(w,r,Y) = 2^{\frac{1}{3}}\, Y^{\frac{4}{3}}\, r^{\frac{1}{3}}\, w^{-\frac{1}{3}}} \Rightarrow \text{igual a } (7)$$

$$\text{y } K(w,r,Y) = \lambda\frac{Y}{4r} \overset{(13)}{=} \frac{\left(Y^{\frac{1}{3}}(2w)^{\frac{2}{3}}(4r)^{\frac{1}{3}}\right)Y}{4r}$$

$$= Y^{\frac{4}{3}} \cdot \frac{w^{\frac{2}{3}}}{r^{\frac{2}{3}}} \cdot \frac{2^{\frac{2}{3}}2^{\frac{2}{3}}}{2^2}$$

$$\Rightarrow \boxed{K(w,r,Y) = \left(\tfrac{1}{2}\right)^{\frac{2}{3}} Y^{\frac{4}{3}}\, w^{\frac{2}{3}}\, r^{-\frac{2}{3}}} \Rightarrow \text{igual a } (6)$$

a4

(A.) 2) f) $a=1 \overset{(e)}{\Rightarrow} C_{lp}(w,r,Y) = \left(\frac{3}{2^{2/3}}\right) Y^{\frac{4}{3}} w^{\frac{2}{3}} r^{\frac{1}{3}}$

$$\Rightarrow \begin{cases} CMe(w,r,Y) = \frac{C_{lp}(w,r,Y)}{Y} = \boxed{\left(\frac{3}{2^{2/3}}\right) Y^{\frac{1}{3}} w^{\frac{2}{3}} r^{\frac{1}{3}}} \\ CMg(w,r,Y) = \frac{\partial C_{lp}(w,r,Y)}{\partial Y} = \frac{4}{3}\left(\frac{3}{2^{2/3}}\right) Y^{\frac{1}{3}} w^{\frac{2}{3}} r^{\frac{1}{3}} \end{cases}$$

$$= \boxed{\left(2^{\frac{4}{3}}\right) Y^{\frac{1}{3}} w^{\frac{2}{3}} r^{\frac{1}{3}}}$$