

SELECTED EXERCISES TOPIC V

7.3.a

SHOW THAT THE PRODUCTION FUNCTION $f(x) = x^\alpha$ WITH $0 < \alpha < 1$ WHERE x IS THE SOLE PRODUCTION FACTOR HAS BOTH DIMINISHING RETURNS TO FACTOR x AND DECREASING RETURNS TO SCALE.

7.5 SHOW THAT THE PRODUCTION FUNCTION $f(L, K) = (\alpha L^p + \beta K^p)^{1/p}$ WHERE $\alpha, \beta > 0$ AND $p \in \mathbb{R}$, HAS CONSTANT RETURNS TO SCALE.

8.2 A FIRM HAS THE FOLLOWING TECHNOLOGY: $f(L, K) = 3L^{1/3}K^{1/3}$

1) FIND THE TECHNICAL RATE OF SUBSTITUTION. BRIEFLY EXPLAIN ITS MEANING.

2) FIND THE RETURNS TO SCALE. WHAT DO THEY MEAN?

3) FIND THE MARGINAL PRODUCTIVITY AND THE AVERAGE PRODUCTIVITY OF BOTH LABOR AND CAPITAL.

4) FORMULATE THE PROFIT-MAXIMIZATION PROGRAM.

5) ASSUMING $P = w = 2, r = 1$, FIND THE LABOR AND CAPITAL DEMANDED QUANTITIES, AS WELL AS THE QUANTITY OF PRODUCTION.

8.4 PART D. GIVEN $f(L, K) = \min\{L, K\}$ AND PRICES P, w, r :

1) DRAW AN ISOQUANT

2) FIND THE FACTOR DEMAND FUNCTIONS AND THE SUPPLY FUNCTION (PROFIT MAXIMIZATION)

3) DERIVE THE LONG-RUN COST FUNCTION (COST MINIMIZATION)

4) GIVEN THIS COST FUNCTION, FOR GIVEN PRICES P, w, r , HOW MUCH PRODUCTION Y WOULD YOU CHOOSE?

5) WHAT IS THE PERCENT INCREASE IN LONG-RUN COSTS IF THE WAGE IS INCREASED BY 1%? COMMENT.

6) OBTAIN THE SHORT-RUN COST FUNCTION

7) SUPPOSE $w = r = \bar{K} = 1$. FOR WHAT LEVEL OF PRODUCTION Y^* THE LONG- AND SHORT-RUN COSTS COINCIDE?

(PRACTICE ALSO WITH $f(L, K) = L + K$)