School Choice and Tiebout

School choice mechanisms with endogenous residential location and peer effects

C.Calsamiglia, F.Martínez-Mora, A.Miralles

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ONGOING RESEARCH

- No draft, no definite results
- Suggestions gratefully acknowledged

School Choice: new questions

- A two-side many-to-one matching problem
- How to assign students to schools
- We incorporate elements not yet analyzed by specialized theoretical literature
  - Endogenous residential location
  - Peer effects
- We provide a theoretical insight on:
  - Effects of assignment mechanisms on urban and inter-school social seggregation
  - Welfare comparison across school choice mechanisms

Endogenous residential location

- E.g. Black (1999)
- Children's schooling is an important residence location factor
- ...if residence priority exists
- Rents separate agents according to willingness to pay

Peer effects

- E.g. Gibbons et al. (2008)
- Parents take children's classmates into account
- Effect on student's performance empirically unclear
- ...however what matters is that parents *perceive* it as important

Seggregation

- Families with more willingness to pay separate from those with less
- ...either in choosing residence (Tiebout) or in choosing school
- ...depending on the chosen school choice mechanism
- ...across districts/schools that are *identical a priori*

Boston Mechanism (BM) and Deferred Acceptance (DA)

- Parents report a ranking over schools. Round by round assignment
- In each round we consider each not-removed student for her reported best school that has not rejected her yet
- With excess demand, schools reject some students according to priorities and lotteries
- Differences with respect to how **accepted** students are treated:
  - BM: they obtain their slots and do not go to further rounds (**definite acceptance**).
  - DA: they are reconsidered for that school in further rounds (**tentative acceptance**).

Example: Agents 1,2,3; Schools a,b,c

Pref.	Prio.	BM	DA
$a \succ_1 b \succ_1 c$	$1 pr_a 2$	Round 1 $1 \rightarrow a, 2 \nrightarrow a, 3 \rightarrow c$	Round 1 $1 \rightarrow a, 2 \nrightarrow a, 3 \rightarrow c$
$a \succ_2 c \succ_2 b$	$2 pr_c 3$	Round 2 $2 \nrightarrow c$	Round 2 $1 \rightarrow a, 2 \rightarrow c, 3 \nrightarrow c$!
$c \succ_3 b \succ_3 a$		Round 3 $2 \rightarrow b$ (put c first)	Round 3 $1 \rightarrow a, 2 \rightarrow c, 3 \rightarrow b$

Summary of the model

- We consider a municipality with three equal-sized districts with one school each
- District 3 hosts the worst school. Districts (and schools) 1 and 2 are identical a priori
- Agents' unidimensional types (willingness to pay) affect school quality
- We assume peer effect related to agent's type: log-supermodularity
- Timing: 0) Voting; 1) Residence location (rent); 2) School choice game
- We study BM and DA with and without residence priorities

Summary of the results

- With residence priorities both BM and DA obtain the same outcome: urban and inter-school seggregation
- Without residence priorities DA cannot generate seggregation
- Instead, BM generates inter-school seggregation (not under log-submodularity)
- Low types vote against residence priorities
- Low and high types tend to prefer BM to DA
- Small "transport costs" induce seggregation even in DA
- Half slots with priority as in Boston: DA generates urban seggregation, BM may not

More formally...

- Districts 1,2,3 and one school in each, capacity 1/3 each
- Rents $r_j, j \in R = \{1, 2, 3\}$: one of them set to 0
- Mass 1 of agents with types $t \sim \Phi : T = [\underline{t}, \bar{t}] \to [0, 1]$
- School quality $q_j, j \in \{1, 2\}$: average t over attendants
- Utility: $h(q, t) - r$
- $h = 0$ if school 3
- h log-supermodular, increasing, continuous, $h(q,\underline{t})$ constant in q

Timing and equilibrium

1. Voting on school choice mechanism (?)
2. Residential market clearing: action $\in R$, $T_j = \{t \in T : R(t) = j\}$
3. School choice game and assignment: action is either rank school 1 first (S_1) or school 2 (S_2)

- Endogenous common beliefs on final school qualities $\hat{q}_1, \hat{q}_2$
- Equilibrium: beliefs $\hat{q}_1, \hat{q}_2$, rents $r_{1,2,3}$ and strategy profile $\sigma(\cdot; \hat{q}_1, \hat{q}_2, r_{1,2,3})$
  - $E(t \,|\, t \in T_j) = \hat{q}_j, j = 1, 2$
  - Given $\hat{q}_1, \hat{q}_2$, $T_{1,2,3}$ and the mechanism, each t is best choosing between $S_1$and S_2
  - Given the school choice equilibrium (random) assignment and $r_{1,2,3}$, each t is best choosing over R
  - The mass of agents choosing district j is $1/3$
- Sequential: $\exists\ \hat{q}_1^n, \hat{q}_2^n \rightarrow \hat{q}_1, \hat{q}_2$ s.t. $\sigma(\cdot; \hat{q}_1^n, \hat{q}_2^n, r_{1,2,3}) \rightarrow \sigma(\cdot; \hat{q}_1, \hat{q}_2, r_{1,2,3})$

Residential Priorities

Unique sequential equilibrium such that
$T_3 = [\underline{t}, a)$, $T_2 = [a, b]$, $T_1 = (b, 1]$. Those students in T_j attend school j.

- $\Phi(a) = 1/3 = 1 - \Phi(b)$
- Both for BM and DA:
  - DA: strategy-proofness
  - BM: residents in district 1 use S_1; knowing that, residents in district 2 use S_2
- Equilibrium rents render types a and b indifferent
- There is an equilibrium with no seggregation ($\hat{q}_1 = \hat{q}_2$ and $r_1 = r_2$) yet it is not sequential

DA with no priorities

No equilibrium such that $\hat{q}_1 \neq \hat{q}_2$

- No priorities $\Longrightarrow$ All rents are zero
- Say $\hat{q}_1 > \hat{q}_2$: since DA strategy-proof, all types submit the same ranking S_1. Accordingly, slots are randomly assigned, thus $\hat{q}_1 = \hat{q}_2$
- There is a sequential equilibrium such that $\hat{q}_1 = \hat{q}_2$ (e.g. everyone using S_1)

BM with no priorities (I)

m_j mass of agents using strategy S_j. Assume $\hat{q}_1 > \hat{q}_2$

- **Case 1: Both schools 1 and 2 give all their slots in the first round ($m_2 \geq 1/3$)**
  - Choose S_1 if $\frac{h(\hat{q}_1,t)}{h(\hat{q}_2,t)} > \frac{m_1}{m_2}$ (S_2 o/w)
- **Case 2: School 2 does not give all its slots in the first round ($m_2 < 1/3$)**
  - Choose S_1 if $\frac{h(\hat{q}_1,t)}{h(\hat{q}_2,t)} > 2$ (S_2 o/w)
- Log-supermodularity $\implies$ LHS increasing in t
  - Equilibrium threshold $\hat{t}$: $\frac{h(E(t \mid t \geq \hat{t}),\hat{t})}{h(E(t \mid t \leq \hat{t}),\hat{t})} = \min\left\{2, \frac{1-\Phi(\hat{t})}{\Phi(\hat{t})}\right\}$

BM with no priorities (II)

$\exists$ equilibrium characterized by $\hat{t}$: types below play S_2, above S_1

- Log-supermodularity not necessary but tight. No such equilibrium if log-submodularity
- $\exists$ equilibrium with $\hat{q}_1 = \hat{q}_2$ yet not sequential
- No urban seggregation. Inter-school seggregation

Welfare comparison?

- Not clear prediction (if any), it depends on h and Φ
- $w_M(t) =$ expected welfare given mechanism, $M \in \{RE, DA, BM\}$
- Remark 1: Consider environments with log-supermodularity and equilibria such that $\hat{t} \geq a$. Then $w_{BM}(t)/w_{DA}(t)$ is U-shaped with minimum at $t = a$, and $w_{BM}(t)/w_{RE}(t)$ is decreasing for $t > b$.
- Remark 2: For all types below a we have $w_{BM}(t) > w_{RE}(t)$ and $w_{DA}(t) > w_{RE}(t)$. Moreover, $w_{BM}(t) > w_{DA}(t) > w_{RE}(t)$ for types sufficiently close to 0.

Robustness: transport costs

- Appart from rent, pay c (small) if attend school other than district's
- Each agent prefers to live in the district where she has more chances to send her child
- Even DA without priorities has an equilibrium with urban seggregation
- Threshold t_c
  - Types below choose district 2 and prefer school 2 to school 1
  - Types above are indifferent among all districts and prefer school 1 to 2
  - Equilibrium rents hold indifference
  - $r_2 = 0 < r_3$!
- Quantitatively not important when c close to 0

Robustness: 50% slots with priority (Boston)

- Assume each agent endowed by unique lottery number
- Priority slots first assigned
- Assume $\hat{t} \geq a$ (case 1 BM)
- Half-slot policy has no effect in BM
- Yet it may generate perfect urban seggregation in DA
- Similar with ε% and priority slots last assigned

An unfinished research work

- Outside option (private school)
- Bidimensional types (income, ability)
- Taxes
- Other than Condorcet
- More...