

## List of Mock Exam Questions

The final exam will have 4 long questions (you might need around half an hour for each long question). Each question has several parts that add up to 2.5 points (ten points in total in the exam.) Each question makes reference to one topic **of the syllabus**. Each question will contain at least one theoretical part and one practice part. I leave this list of questions that might be parts of questions in the exam, so that you get an idea about the type of problems you will have to solve.

1. Define indirect utility. When do we say that indirect utility has the Gorman form? Elaborate an example of a utility function that implies the Gorman form? What effects does the Gorman form have regarding the problem of aggregating individual demands?
2. Show that strict monotonicity implies local non-satiation whereas the converse (the latter implies the former) is not always true.
3. What does the Slutsky equation say about the sign and size of the income effect for a Giffen good? No formulas, no more than twenty words.
4. Show that the indirect utility function is quasi-convex. That is, when evaluated at some intermediate point between two extreme reference points, it is never higher than the maximum indirect utility of the extremes. (You may assume continuous preferences and LNS)
5. Suppose that consumer's preferences are lexicographic in goods 1 and 2 (where good 1 is more important than good 2) and that the consumer space is  $\mathbb{N} \times \mathbb{R}_+$  (the Cartesian product of the set of natural numbers and the set of positive real numbers.) Are these preferences representable by a utility function? Show impossibility or else find a utility function that represents these preferences.
6. In one short paragraph, in what sense aggregating supplies is easier than aggregating demands?
7. In which part of the proof of Arrow's representation theorem continuity of preferences play a key role?
8. Define the axiom that makes sense in the context of choice under uncertainty and it does not in when choosing between sure outcomes. Explain why it makes sense in the first context.
9. If  $u$  and  $v$  are utility functions representing identical preferences over lotteries, what is the relation between them? What would your answer be if we talked about preferences over consumption bundles? When do we talk about ordinality and about cardinality, and what does it mean?
10. State and prove Hotelling's lemma when we have only one output and  $n-1$  inputs and the production function is differentiable everywhere.
11. Is the expenditure function concave or convex in  $p$ ? Show it.
12. Prove that when the set of alternatives is finite, all rational preferences are representable by a utility function.

13. Prove that when choices are rationalizable, then the preferences rationalizing these choices are the revealed preferences.

14. Thanks to a satisfaction survey, an econometrician was able to estimate individual satisfaction level as a differentiable function of income and prices:  $v_i = v_i(p, w_i)$ . He sadly reported he got no data on individual demands. What can you tell him to make his day happier?

15. What is the relation between WARP and rationalizability when  $X$  is uncountable, and what conditions on the data about agent's taken choices we need?

17. Put an example in a graph showing that individual WARP does not imply aggregate WARP?

18. Define WARP. Define WAPM.

19. What is the relation between WARP and the law of compensated demand? Show it.

20. What is a support function of a convex set? State the duality theorem. What does it say about the relation between the Hicksian demand and the expenditure function?

21. Explain an experimental result that casts doubt on the independence axiom when choosing among lotteries. (In one paragraph)

22. Derive the definition of relative risk aversion from the absolute risk aversion in a small lottery over percentage gains/losses. What problem does it solve with respect to absolute risk aversion?

23. If a Bernoulli utility function  $u$  is more absolute-risk averse than  $v$ ,  $u$  is a concave transformation of  $v$ . True or false? Prove it or show a counterexample.

24. For the linear utility function  $U = x + y$  find the demand correspondence, the indirect utility function, the expenditure function, and the Hicksian demand. Then, for  $p_y = 2$  and  $w = 60$  find the compensating variation, the equivalent variation and the change in consumer surplus if  $p_x$  changes from 3 to 1.

25. Show that a continuous preference is homothetic if and only if it is represented by a utility function that is homogeneous of degree 1.

26. Knowing that the (assumed differentiable) Walrasian demand  $x(p, w)$  is homogeneous of degree zero, prove that  $D_p x(p, w) \cdot p + D_w x(p, w) w = 0$ .

27. Consider the utility function  $u(x) = x_1 + a_2 \ln x_2 + a_3 \ln x_3$ , where  $a_2, a_3 > 0$ . Under which conditions do we have  $x_1(p, w) > 0$ ? Calculate the indirect utility function in such a case.

28. Prove that the Slutsky matrix is the Hessian (matrix of second derivatives) of the expenditure function (assuming  $x(p, w)$  differentiable).

29. A regulatory agency needs to know the cost function of a regulated firm that produces an unidimensional output  $q_t$ . It observes the firm's demand for inputs  $z_t$ , the output  $q_t$ , and prices

for inputs  $w$  for a large number of months. Assuming that the production technology does not evolve over time, how would you help the regulatory agency in its mission?

30. Illustrate in a graph that any efficient aggregate assignment is aggregate profit maximizing for some nonnegative prices  $p$  if all individual feasible sets are convex.

31. Let preferences over lotteries  $L$  satisfy rationality, continuity and the independence axiom. According to these preferences let  $L_{\max}$  and  $L_{\min}$  be the most- and least-preferred lotteries, respectively. Define the function  $U(L)$  implicitly as  $U(L) L_{\max} + (1-U(L)) L_{\min} \sim L$ . Prove linearity of  $U(L)$ , that is,  $U(aL+(1-a)L')=aU(L)+(1-a)U(L')$ .

32. An individual whose wealth is  $W > 0$  could have an accident in which she loses  $L$  with probability  $\pi$ . Let  $u$  denote the individual utility for money. Assume this function is differentiable at least twice with  $u' > 0$  and  $u'' < 0$ . An insurance policy specifies the premium the consumer has to pay regardless of the accident ( $P$ ), and the amount the insurance company reimburses if the accident occurs ( $Z$ ). The latter is chosen by the consumer. Show that if insurance is actuarially fair (i.e.  $P = \pi L$ ), the consumer will insure the entire loss ( $Z=L$ ).