Decision making under risk. Part 2

1. Consider two states of the world $\Omega = \{s_1, s_2\}$. An agent has preferences over state-contingent (monetary) payoffs $\mathbf{x} = (x_1, x_2) \in \mathbb{R}^2_+$. The price of consumption in state s_1 is normalized to 1 such that the market value of a bundle $\mathbf{x}$ is $x_1 + px_2$. Let the budget be w. The agent's preferences can be represented by a utility function $U(\mathbf{x})$.

a. Consider first the standard case in which an agent trusts her probability estimate $q = Pr(s_1) = 3/4$ and $(1-q) = Pr(s_2) = 1/4$. U satisfies expected utility and we have $U(\mathbf{x}) = q\log(x_1) + (1-q)\log(x_2)$. Show that the Walrasian demand function is $\mathbf{x}^*(p, w) = \frac{w}{4p}(3p, 1)$.

b. Depict the tangency-situation graphically, using that budget hyperplanes have normal vectors $(1, p)$ and preferred sets are supported by hyperplanes whose normal is $\nabla U(x_1, x_2) = (qu'(x_1), (1-q)u'(x_2))$.

c. From now on, we consider the case in which the agent does not have much confidence in her probability estimate of $1/2$, and she considers instead a range of plausible $q \in [\frac{1}{4}, \frac{3}{4}]$. She wants to make consumption plans whose expected utility is fairly robust to the unknown probability q. More precisely, her utility function is

$$V(\mathbf{x}) = \min_{q\in[\frac{1}{4},\frac{3}{4}]} qu(x_1) + (1-q)u(x_2),$$

with u increasing and concave. Find the minimizing probabilities as a function of $\mathbf{x}$:

$$q^*(\mathbf{x}) = \arg\min_{q\in[\frac{1}{4},\frac{3}{4}]} qu(x_1) + (1-q)u(x_2).$$

d. Show that a typical indifference curve has a kink on the 45-degree line. Use that the supporting hyperplanes have normal vectors $(q^*(\mathbf{x})u'(x_1), (1 - q^*(\mathbf{x}))u'(x_2))$ at any point $\mathbf{x}$.

e. Now assume $u(x) = \log x$. First, show that $p = 1$ implies $\mathbf{x}^*(1, w) = \frac{w}{2}(1, 1)$.

f. Next, show that $p \in [\frac{1}{3}, 3]$ also implies full insurance $\mathbf{x}^*(p, w) = \frac{w}{1+p}(1, 1)$.

g. Finally, show that $p < \frac{1}{3}$ implies $\mathbf{x}^*(p, w) = \frac{w}{4p}(3p, 1)$.

2. Consider the family of vN-M functions

$$\mathcal{U} = \{u : [0, b] \to [0, b] | u(x) = \lambda \min\{x, 5\} + (1-\lambda)x \text{ for some } \lambda \geq 0\}.$$

a. Characterize the relation $\succsim_S$ on distribution functions with support in $[0, b]$ which satisfies that $F \succsim_S G$ if and only if $\int_0^b u(y)dF(y) \geq \int_0^b u(y)dG(y)$ for all $u \in \mathcal{U}$.

b. Now consider the special case of F, G with $F(5) = G(5)$. Try to restate the condition you found in (a) in terms of (conditional) expectations.

c. Is this stronger or weaker than FSD, SSD?

Optional, but recommended (not required to hand in)

3. Solve 6.E.1 in MWG.

4. Find an example of two increasing (resp. increasing and concave) utility functions which disagree on the ranking of two distributions F, G that cannot be ranked by FSD (resp. by SSD).

5. Consider two probability density functions f and g, strictly positive on their common support $[a, b]$. We say that f dominates g according to the *monotone-likelihood-ratio* order (MLR) if, for all $x, y \in [a, b]$, we have $f(x)/g(x) \geq f(y)/g(y)$ whenever $x \geq y$. Show that MLR is a special case of FSD. *Hint*: Rearrange, then integrate the inequality twice.