Decision making under risk. Part 1

1. Let $\succsim$ satisfy completeness, transitivity, and the Independence axiom on a set Π. Prove that for any two alternatives $x, y \in \Pi$ with $x \succsim y$ and for any $1 > \alpha > \beta > 0$:

$$\alpha x + (1-\alpha)y \succsim \beta x + (1-\beta)y.$$

2. Consider an agent whose preferences satisfy the Independence Axiom.

a. Consider four lotteries $p, q, r, s \in \Delta(X)$ over prizes in $X = \{x, y, z\}$ with $p = (p(x), p(y), p(z))$, etc.

- $p = (0.2, 0.3, 0.5)$,
- $q = (0.25, 0.35, 0.4)$,
- $r = (0.8, 0, 0.2)$,
- $s = (0.9, 0.1, 0)$.

When you learn $p \succsim q$, what can you infer about the ranking of r relative to s?

b. For the same lotteries, suppose that sure prizes can be ranked such that $\delta_z \succsim \delta_y \succsim \delta_x$. Show that $p \succsim_{FSD} q$.

c. Verify that the Independence axiom implies a preference for FSD-dominant lotteries by showing that the axiom indeed implies $p \succsim q$.

3. Determine whether the following utility criteria satisfy the axioms of expected utility:

1. Preference for "greater certainty": $v(p) = \max_{x \in X} p(x)$.
2. The agent considers a subset $G \subseteq X$ "good" outcomes. He ranks lotteries by the total probability of a good outcome: $v(p) = \sum_{x \in G} p(x)$.
3. Judge by worst case: $v(p) = \min_{x \in X}\{u(x) | p(x) > 0\}$.
4. Judge by most likely prize: $v(p) = \arg\max_{x \in X} p(x)$.

4. Suppose two EU maximizers with von Neumann-Morgenstern utility functions u_1 and u_2 with $u_2 = \phi \circ u_1$.

a. Show that $\phi' > 0, \phi'' < 0$ implies that at all wealth levels w the degree of absolute risk aversion of 2 is greater than that of 1.

b. Show that $\phi' > 0, \phi'' < 0$ implies that 2 is more risk-averse in the sense of Arrow and Pratt.

Recommended Exercise. (No need to hand in)

5. Consider an EU maximizer with vNM function $u(x) = 2\sqrt{x}$ and a fair coin flip. If heads show up she gets 71, if tails show up she gets 15.

a. Determine the risk premium associated to this gamble at wealth level 10.

b. Calculate the degrees of absolute and relative risk aversion at wealth levels w. Would the risk premium change if wealth decreased to 1?