Supply. Part 1

1. Study properties of the production set.

(a) Show that the possibility of inaction together with convexity imply non-increasing returns to scale.

(b) Show that if Y is convex and has constant returns to scale, then it is additive.

(c) Show that if Y is additive and has constant returns to scale, then it is a convex cone.

2. Try to prove that if the production set Y is closed, convex, and satisfies free disposal, then it can be recovered from the profit function alone. Do it in the following way: define

$$\hat{Y} = \{y \in \mathbb{R}^n : py \leq \pi(p) \text{ for all } p \in \mathbb{R}^n_+\}$$

(a) Show that $Y \subset \hat{Y}$ (Trivial).

(b) Show that $\hat{Y} \subset Y$ (Easy).

[Hint: Use a separating hyperplane argument to show that any $x \notin Y$ must lie outside of $\hat{Y}$. Free disposal helps you deal with the non-negativity of p.]

3. Consider the following three "two inputs-one output" technologies $f(z) = z_1 + z_2$, $g(z) = min\{z_1, z_2\}$ and $h(z) = [z_1^\rho + z_2^\rho]^{1/\rho}$ with $\rho < 1$.

(a) Check whether the above technologies are additive.

(b) Find the cost functions $c(w, q)$ and factor demands $z(w, q)$.

4. Solve problem 5.D.3 in MWG.

Recommended Exercise. (No need to hand in)

5. Proposition 5.C.2. in MWG links properties of cost functions $c(w, q)$ and production functions $f(z)$, where z denotes inputs. Prove properties (i)-(vii).